Scientifically grounded Hypothesis on the Existence of a physical fifth Dimension

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ABSTRACT

For euclidean space a transfer of the equation for a circle $x^2+y^2=R^2$ and the equation for the surface of a sphere $x^2+y^2+z^2=R^2$ to four spatial dimensions will be performed: $x^2+y^2+z^2+u^2=R^2$. Analogous to the surface of a sphere one receives a curved three-dimensional riemannean space which is unlimited and cyclic. In all directions of this space one gets back to the starting point after moving a distance of $n*2\pi R$. This space is finite, because its volume is $2\pi^2R^3$. With time as another dimension in this "surface of a four-dimensional sphere" we receive a metric which is similar to the cosmological Robertson-Walker-Model. An additional time-dependence of radius $R(t)=v_0*t$ leads to a Hubble constant which is indepentent of v_0 and which reciprocal value is equal to expansion time t, the age of the universe.

1 Geometry historically

Geometry in its original meaning was practised in old Egypt as land-surveying for repartitioning of fields after the annual flooding by the river Nil. In the 6th century B.C. the Greek took charge of it and developed it to a theory of spatial figures. Euclid teached in Alexandria around 300 B.C. his axiom of parallels, which meant that for a given point which is not placed on a given line there is only one parallel line through this point.

After 2100 years of development by Chinese, Indians and Arabians, when the euclidean Geometry had become a part of mathematics, the end of the euclidean Geometry began with Gauß, who did not see any reason why there should not be curved spaces, in which there exists more than one parallel through a point not on a given line.

In 1832 when Gauß's school friend Bolyai consulted him because of the work of his son on a non-euclidean geometry of funnel-shaped negativ curved surfaces Gauß did not dare to publish that. The publication of a non-euclidean hyperbolic geometry was done short after this by Lobatchewsky.

After the ice was cut, Gauß supported his student Riemann, who outlined an abtract geometry of curved multi-dimensional spaces. They were not determined by axioms, but by inner measurement. Important terms for riemannean spaces are metric, line element, general coordinate system, geodetic lines, Gauß curvature, riemann curvature tensor and general coordinate transformation.

These visionary mathematical abstractions of our common three-dimensional euclidean space got a new real physical significance by Einstein's work. He postulated the equvalence principle[1] which lead to a description of gravity as a general coordinate transformation between accelerated systems. This means a curvature of spacetime. Einstein connected curvature of spacetime with its field sources, the mass distribution, by his gravity equation.

2 Surface of a four-dimensional Sphere

In addition to the common three spatial dimensions we take a fourth spatial dimension. Mathematically that is not a difficulty. As it is possible to cut a cube with ist coordinates x,y,z into x,y-films along its height z, we may imagine cutting our four-dimensional space along its fourth axis u into three-dimensional x,y,z-layers. The result is an infinte number of parallel three-dimensional spaces, each belonging to a different value of u. Parallel spaces are mathematically not a problem. In the following we use the term 'heigh' for the fourth coordinate u.

Now we take a look at all points with coordinates x,y,z,u which have the same four-dimensional distance R from the coordinate system origin (0,0,0,0):

$$x^2 + y^2 + z^2 + u^2 = R^2 = const. \tag{1}$$

This is the transfer of the circle equation and sphere equation to four dimensions and thus we call this set of points determined by this equation the 'surface of a four-dimensional sphere'.

This set of points is also a coherent three-dimensional layer. For example the 'north pole', the point with the maximum heigth $u = R$ and x=0, y=0, z=0, has neighbouring points in three different directions: x, y and z. These neighbouring points are also in a distance R from origin, but they are lying on three different coordinate axes. The layer is cuved, because for each of this neighbouring points R is a little slanted, x, y and z are no longer zero, height u is smaller than R and the coordinate axes, which are rectangular to R (tangents), are slanted against the coordinate axes in the north pole.

In the curved layer one can move around the origin in a constant distance R from the origin. That is the reason, why we can call it the surface of a four-dimensional sphere. In every point of the layer there exist three orthogonal

[1] equivalence of inertia and gravity, mass and energy, free falling system and system hoovering in space

directions in which one can move around the origin on a circle within the layer. Thus it builds up a three-dimensional curved space. In the following we will use the term 'surface' for it, and for the 'sphere' we neglect the specification 'four-dimensional'.

3 Metric in the Surface of the Sphere

Eq.(1) as shown up is the pythagoras for a pentagon in four dimensions of euclidic space. An analogous defintion of the same space as a riemannean space is done by definition of the line element ds. Then R is received by integration over ds from the origin to the point in the surface:

$$ds^2 = dx^2 + dy^2 + dz^2 + du^2 \tag{2}$$

In the following we will set up the metric for the curved three-dimensional space. First of all we have to substitute the variable u (section 3.1). Then it is neccessary to change to polar coordinates (section 3.2) to represent the metric by parameters of the surface itself (section 3.3). After that we are able to compute the volume of the curved three-dimensional space (section 3.4).

3.1 Substitution of u by x, y, z and R

We want to represent points of constant four-dimensional distance R from origin with parameters of the surface itself. For that we substitute height u with eq.(1) by x, y, z and R:

$$u = \sqrt{R^2 - x^2 - y^2 - z^2} \tag{3}$$

By this substitution we use the points of the equatorial x,y,z-plane as parameters, each having height $u = 0$. For each of these points we can compute a height u by eq.(3) to get a point on the surface. To every point in the equatorial plane we get two points on the surface, one on the northern hemisphere and one on the southern hemisphere.

This substitution helps us to build up the metric in the surface of the sphere, which allows us to compute the distance between two points on the surface (following its curved structure) by integration. By derivations of eq.(3) one gets:

$$du = -\frac{xdx + ydy + zdz}{\sqrt{R^2 - x^2 - y^2 - z^2}} \tag{4}$$

With this equation we can substitute du in eq.(2) for the line element:

$$ds^2 = dx^2 + dy^2 + dz^2 + \frac{(xdx + ydy + zdz)^2}{R^2 - x^2 - y^2 - z^2} \tag{5}$$

This is the metric of a three-dimensional space with constant positive curvature paramtrized by points of the equatorial plane. Near the north pole, where

$x = 0$, $y = 0$, $z = 0$, the last term vanishes and we get the pythagoras for the line element, the same as in the origin. For larger x,y,z we get a larger ds then we compute after pythagoras, because the distance on the surface then is larger than its projection on the equatorial plane as we move down in the surface from 'north pole' with $u = R$ to the equatorial plane with $u = 0$.

3.2 Change to polar coordinates

To describe the metric by parameters of the curved three-dimensional space itself, we have to change to polar coordinates ρ, θ, ϕ within the equatorial plane:

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \arccos\left(\frac{z}{\rho}\right), \quad \phi = \arcsin\left(\frac{y}{x}\right) \tag{6}$$

which results in $x = \rho \cos\phi \cos\theta$, $y = \rho \sin\phi \cos\theta$, $z = \rho \sin\theta$. For polar coordinates one receives for the line element $d\overline{s}$ in the equatorial plane:

$$d\overline{s}^2 = dx^2 + dy^2 + dz^2 = d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{7}$$

Eq.(4) in polar coordinates becomes:

$$du = -\frac{xdx + ydy + zdz}{\sqrt{R^2 - x^2 - y^2 - z^2}} = -\frac{\rho d\rho}{\sqrt{R^2 - \rho^2}} \tag{8}$$

Eq.(7) and eq.(8) inserted in eq.(5) gives:

$$ds^2 = d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{(\rho d\rho)^2}{R^2 - \rho^2} \tag{9}$$

and after some transformation:

$$ds^2 = \frac{(d\rho)^2}{1 - \rho^2/R^2} + \rho^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{10}$$

That is eq.(5) in polar coordinates.

3.3 Metric of the three-dimensional curved space

With a polar coordinate system we are able to describe the metric of the curved three-dimensional space by parameters of the surface itself. We move from the equatorial plane to the surface by projecting each point of the equatorial plane along the heigt u after eq.(3) onto the surface. Starting from the 'north pole', the point with maximum height $u = R$, we may take over the coordinates θ und ϕ we used in the origin $u = 0$, $x = 0$, $y = 0$, $z = 0$:

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \arccos\left(\frac{z}{\rho}\right), \quad \phi = \arcsin\left(\frac{y}{x}\right) \tag{11}$$

A modification is only neccessary on the radius coordinate. With R and ρ height u is determined after eq.(3), with it also the angle χ between the radii and also arc a in the surface from 'north pole' to the point of study:

$$\sin\chi = \frac{\rho}{R}, \qquad a = \chi R \tag{12}$$

Which means

$$\chi = \arcsin\left(\frac{\rho}{R}\right)$$

$$\frac{d\chi}{d\rho} = \frac{1}{\sqrt{1-\frac{\rho^2}{R^2}}}\frac{1}{R}$$

and results in the following substitutions

$$R^2 d\chi^2 = \frac{d\rho^2}{1-\frac{\rho^2}{R^2}}$$

$$\rho^2 = R^2\sin^2\chi$$

This substituted in eq.(10)

$$ds^2 = R^2(d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

leads to the metric of polar coordinates with parameters of the surface itself, which was to find:

$$ds^2 = da^2 + R^2\sin^2\frac{a}{R}\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Arc a is in the three-dimensional space similar to distance r:

$$ds^2 = dr^2 + R^2\sin^2\frac{r}{R}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{13}$$

The factor $\sin\frac{r}{R}$ together with R can be viewed as radius ρ_u which we receive when we move around the 'noth pole' in a constant distance r and divide the measured circumfence by 2π. It is the projection of the curved distance r onto the equatorial plane. In the beginning it increases as r, reaches a maximum at $\sin\frac{r}{R} = 1$ which means $r = \frac{\pi}{2}R$ (equator), and then decreases until for $r = \pi R$ the projection ρ_u again is zero. With $r = \pi R$ half an orbit is done, with $r = 2\pi R$ a full orbit arount the origin is done.

3.4 The volume of the curved three-dimensional space

The volume of a spherical shell in a distance r of thickness dr is

$$dV = 4\pi R^2\sin^2\left(\frac{r}{R}\right)dr \tag{14}$$

This integrated over $R\,d(r/R)$ for all possible spherical shells form 0 to π results in:

$$V = 4\pi R^2\left(\frac{\pi}{2}\right)R \tag{15}$$

Thus we get for the volume of the curved three-dimensional space with constant curvature radius R :

$$V = 2\pi^2 R^3 \tag{16}$$

4 Solution of Einsteins's gravity equation for constant spatial mass density

Einsteins's gravity field equation combines curvature of spacetime with the present masses. If the mass distribution is homogenous and static, then the curvature of space will be given by only one value: the curvature radius. The solution of Einstein's gravity equation for a homogenous, isotropic and static mass distribution is called Robertson-Walker-Metric (RWM):

$$ds^2 = -c^2dt^2 + dr^2 + R^2 \sin^2 \frac{r}{R} \left(d\theta^2 + \sin^2 \theta d\phi^2\right) \tag{17}$$

The only difference between RWM and the metric developed in the previous chapter is the term $-c^2t^2$. This leads to the conclusion, that the RWM with over all space constant curvature radius may be equivalent to a three-dimensional surface of a four-dimensional sphere.

If there are no forces, which hold the masses in distance like the thermal pressure in stars then the mass ditribution will not be stable. Because of gravity the masses will fall in and collapse to a black hole. RWM is used to describe this process in eigen time t. The curvature radius becomes time-dependent and a scaling factor for the whole metric.

$$ds^2 = -c^2dt^2dr^2 + R^2(t) \sin^2 \frac{r}{R(t)} \left(d\theta^2 + \sin^2 \theta d\phi^2\right) \tag{18}$$

So the masses retain always at their coordiates where they have been before (χ, θ, ϕ). Changes are described only by change in the scaling factor $R(t)$.

On the other hand with an increasing $R(t)$ we are able to describe an expanding space. When $R(t)$ increases with eigen time t, then the three-dimensional surface will expand like the surface of an air ballon which is inflated. A RWM with increasing $R(t)$ is a good model for the expansion of our universe. With it are fitting well the observed a) relative velocities of galaxies, b) the red shift caused by this motion and c) the Hubble principle.

The circumfence $2\pi R$ also increases with R. Is there only a little change in $R(t)$ with time, so there will be only little relative velocities an light will be able to perform a full orbit of $2\pi R$ around the universe and will reach in time $T = 2\pi R/c$ the starting point again. The whole universe could be in interaction.

If we have a fast change of $R(t)$ with time of $c/2\pi$, the a point in distance $D = 2\pi R$ will have relative velocity c and therfore will vanish with infinite red shift at the observation horizon.

With faster change of $R(t)$ with time the observation horizon would be in a smaller distance and the part behind it has not any interaction with the starting point.

If the change of $R(t)$ with time is constant $R(t) = wt$, there must have been a time $t_0 = 0$ where also the radius was $R = 0$. Thus time t becomes the age of the universe. The expansion of the circumfence $2\pi w$ divided by the circumfence $2\pi R(t) = 2\pi wt$ results in the Hubble constant: $\frac{2\pi w}{2\pi wt} = \frac{1}{t} = H_0$

With $H_0 = 75$ km/sec per mio parsec (1 parsec = 3,26 light years) we receive a time t of ca. 15 billion years. But the increase of $R(t)$ will probably not be constant, because it is slowed down by gravity forces.

5 Conclusion

The correspondence of the spatial part of the Robertson-Walker-Metric with the metric we developed for the three-dimensional surface of a four-dimensional sphere offers the possibility of an universe, which is similar to a closed, finite and cyclic surface of an expanding air ballon. It would have a volume of $2\pi^2 R^3$ with a radius R of ca. 15 billion light years. This universe can be imagined contained in a five-dimensional euclidean space x, y, z, u (sphere) and time t.